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On the string coupling in a class of stringy orbifold GUTs

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Abstract

In this short note, we examine the relationship between the string coupling constant, g_{STRING}, and the grand unified gauge coupling constant, α_{GUT}, in a highly successful class of models based on anisotropic orbifold compactifications of the weakly coupled heterotic string. These models represent a stringy embedding of SU(6) gauge-Higgs unification in a five dimensional orbifold GUT. We find that the requirement that the theory be perturbative provides a non-trivial constraint on these models. Interestingly, there is a correlation between the proton decay rate (due to dimension six operators) and the string coupling constant in this class of models. Finally, we make some comments concerning the extension of these models to the six (and higher) dimensional case.

String theory is potentially a theory of everything; however, it is still an open question as to whether or not the standard model is one of the effective field theories which lies in the "landscape" of possiblities. The weakly coupled $\mathrm{E}_8 \otimes \mathrm{E}_8$ heterotic string is an excellent framework for obtaining effective low energy field theories with many of the phenomenological properties of the minimal supersymmetric standard model (MSSM). Most recently, several detailed "benchmark" string models with 3 families of quarks and leptons, and one pair of Higgs multiplets have been obtained [1–5]. Additional vector-like exotics and U(1) gauge symmetries decouple; the Yukawa couplings for quarks and leptons are non-trivial and the top quark Yukawa coupling and GUT coupling are equal at the GUT scale (i.e. $y_{top}(M_{\mathrm{GUT}}) = g_{\mathrm{GUT}}$) due to the property of gauge-Higgs unification. In addition, the models have an exact R-parity, and a D_4 family symmetry under which the two light families transform as a doublet, and the Higgs and third family transform as singlets. This latter property might be crucial for generating a hierarchy of quark and lepton masses, while ameliorating the supersymmetric flavor problem.

In a recent paper [6] it was shown how this highly successful class of string models [1–5] could accommodate gauge coupling unification in a 5D orbifold GUT limit. Given the exotic matter content of the two benchmark models outlined in Reference [5], we found (using an effective field theory analysis) 252 ways to achieve unification by varying the cutoff M_{STRING} in the effective field theory, the compactification scale M_{C}, and (most importantly) the spectrum of "light" exotics with mass M_{EX}. Of the 252 different solutions found, 48 were not already ruled out by current (dimension six) proton decay bounds. By assigning VEVs to MSSM singlets, we were able to show how one could realize one of these solutions in the "Model 1A" of Reference [5]. In addition, the solution described in [6] satisfies the constraints for unbroken low energy supersymmetry: $F = D = 0$. This latter feature is essential if we are to understand the origin of the hierarchy between the electroweak and Planck scales.

In this paper we address the important question of whether any of these constructions are consistent with a perturbative string expansion. We find a simple formula for the 10D string coupling g_{STRING} (see Eqn. 6) and show that the constraint $g_{\mathrm{STRING}} < 1$ is correlated with the longevity of the proton. Of course, this result applies only to a very small, even minuscule, portion of the string landscape; however, the relevant question is whether or not it is applicable to those very constrained portions of the string landscape where the minimal supersymmetric standard model can be shown to reside.

The models of Reference [5] are derived from an orbifold compactification of the weakly coupled heterotic string: formally $T^6/\mathbb{Z}_6$-II, which can be parameterized by the root lattice $\mathrm{G}_2 \times \mathrm{SU}(3) \times \mathrm{SO}(4)$. By varying the VEVs of the T (size) and U (shape) moduli associated with the SO(4) lattice, it was shown in References [7–9] that one can achieve a stringy embedding of the highly successful orbifold GUT picture [10–13]. In the literature, this has been called "anisotropic" string compactification [7, 9, 14–17]. The problem, of course, is that the GUT coupling constant in the effective four dimensional theory is proportional to the ten dimensional Yang-Mills coupling (and thus the string coupling, g_{STRING}) by a factor of one over the volume of the six-dimensional compactification. The requirement of acceptable unification in the low energy effective field theory may be inconsistent with the requirement that the underlying string theory be weakly coupled ($g_{\text{STRING}} \lesssim 1$), depending on the precise relationship between the two parameters.

By demanding that the underlying heterotic string theory still be perturbative (i.e., weakly coupled), we show how one can further constrain the parameter space of our models—in fact, of the 252 solutions which were found in Reference [6], only 28 of them turn out to have $g_{\text{STRING}} < 1$, see Table 1 on page 10. Moreover, all of these 28 models have a long lived proton, with $\tau(p \to \pi^0\ e^+) \gtrsim 10^{34}$ y. Because the proton lifetime is proportional to the fourth power of the compactification scale, and the string coupling g_{STRING} is inversely proportional to the volume of the compact space, there is a correlation between M_{C}, g_{STRING} and M_{STRING}, which we make explicit. This means that the question of weak string coupling is not entirely decoupled from the low energy phenomenology in these models. In fact, for a reasonable choice of parameters, a long lived proton seems to be *synonymous with* weak string coupling. A particularly interesting detail is that the same example which we constructed in Section 4 of Reference [6] will survive this round of scrutiny, with $g_{\text{STRING}} \sim 0.5$. In addition, all but one of the nine models which were categorized as "interesting" (see Table 9 in Reference [6]) are eliminated when we require the string coupling to be small. Thus the requirement that we be in a perturbative regime of the underlying string theory gives a new, non-trivial constraint on the "mini-landscape" models. In light of this requirement, we comment on the ability to interpret the models of References [1–5] as six (and higher) dimensional orbifold GUTs.

The String Coupling.—In a given string compactification, the string coupling is set by the VEV of a scalar field, called the dilaton. In general, one has $g^2_{\text{STRING}} \sim e^{2\phi}$. In order to find the exact relationship, one must start from

the ten dimensional effective action for the weakly coupled heterotic string and compactify on some six dimensional manifold. The four dimensional effective action is [18]

$$\mathcal{S}_{eff} = -\int d^4x\sqrt{g}e^{-2\phi}V_6\left\{\frac{4}{\alpha'^4}R + \frac{1}{\alpha'^3}\mathrm{Tr}\ F^2 + \cdots\right\}. \tag{1}$$

where ϕ is the (ten dimensional) dilaton, V_6 is the volume of the compactification, and α' is the parameter which sets the string tension. We can identify the coefficient of the gravity term with Newton's constant:

$$\frac{4e^{-2\phi}V_6}{\alpha'^4} \equiv \frac{1}{16\pi G_N} \Rightarrow G_N \equiv \frac{\alpha'^4 e^{2\phi}}{64\pi V_6}, \tag{2}$$

and the coefficient of the gauge kinetic term with the (four dimensional) Yang-Mills coupling constant[1]:

$$\frac{e^{-2\phi}V_6}{\alpha'^3} \equiv \frac{1}{2g^2_{\mathrm{GUT}}} \Rightarrow \alpha_{\mathrm{GUT}} \equiv \frac{\alpha'^3 e^{2\phi}}{8\pi V_6}. \tag{3}$$

The parameter α' is related to the cutoff in the effective field theory [6, 19]: $\Lambda^{-2} \equiv M^{-2}_{\mathrm{STRING}} \approx \alpha'$. Note that this parameter was chosen in such a way as to capture the maximum amount of stringy (threshold) effects in the low energy effective field theory without actually calculating them [19]. Of course, the exact relationship between α' and M_{STRING} depends on the regularization scheme (see for example [20]). In particular, we will take the standard definition of the *string length* ℓ_{S}, such that it is related to the cutoff by $\ell_{\mathrm{S}} \equiv \frac{\sqrt{\alpha'}}{2} \approx \frac{1}{2M_{\mathrm{STRING}}}$. Finally, the compactification scale is given in terms of the radius of the fifth dimension: $\ell_{\mathrm{C}} = R \equiv \frac{1}{M_{\mathrm{C}}}$.

By exploiting the duality between the $\mathrm{E}_8 \otimes \mathrm{E}_8$ heterotic theory and heterotic-M theory, Hebecker and Trapletti argued [16] that the proper relationship between the 10D dilaton and the string coupling constant is given

[1] Note that we have normalized the gauge fields such that in the fundamental representation of $\mathrm{SU}(N)$ we have $\mathrm{Tr}\ (T_a T_b) = \frac{1}{2}\delta_{ab}$, which is the standard normalization used for phenomenology. In addition the GUT coupling α_{GUT} is evaluated at the string scale M_{STRING}.

by[2]

$$g_{\text{STRING}}^2 \equiv \frac{8e^{2\phi}}{(2\pi)^7}. \tag{4}$$

This gives us a relationship between the (four dimensional) GUT coupling constant at the string scale and the string coupling. By eliminating the dilaton dependence between Equations (3) and (4) we find

$$\alpha_{\text{GUT}} = \frac{\alpha'^3 (2\pi)^6}{2^5 V_6} g_{\text{STRING}}^2 \tag{5}$$

Taking five directions compactified at the string length, ℓ_{S}, and one direction compactified at ℓ_{C}, we find

$$g_{\text{STRING}}^2 = \alpha_{\text{GUT}} \frac{M_{\text{STRING}}}{M_{\text{C}}}. \tag{6}$$

Note that it is entirely possible that the effective field theory be weakly coupled, but that the underlying string theory be strongly coupled.

Are We Perturbative?—Using the relationship in Equation (6), we can examine the 252 different solutions found in Reference [6]. The results of this analysis are shown in Figure 1. Of the 48 models which were not eliminated previously because of dimension six proton decay, 28 have $g_{\text{STRING}} \lesssim 1$. There is certainly a preference for strong coupling in these models: this is a competing effect between the ratio of the string scale to the Planck scale (which sets α_{GUT}) and the ratio of the string scale to the compactification scale (which sets g_{STRING}).

In general, however, it is significant that only the models with long lived protons have small string coupling. As discussed in [6] the proton lifetime scales as $({M_{\text{C}}}^4/\alpha_{\text{GUT}}^2)$. Then using the relation between α_{GUT} and the Planck scale

$$\alpha_{\text{GUT}}^{-1} = \frac{1}{8}\left(\frac{M_{\text{PL}}}{M_{\text{STRING}}}\right)^2 \tag{7}$$

obtained by combining Equations (2) and (3), and the dimension 6 operator contribution to the proton decay rate (see Reference [6]), we obtain the

[2]They showed that for $g_{\text{STRING}} < 1$, the lowest lying massive state is a perturbative heterotic string state, while for $g_{\text{STRING}} > 1$ it is a Kaluza-Klein mode of M theory. At the present time, this is the best estimate we know of for defining the perturbative heterotic string regime.

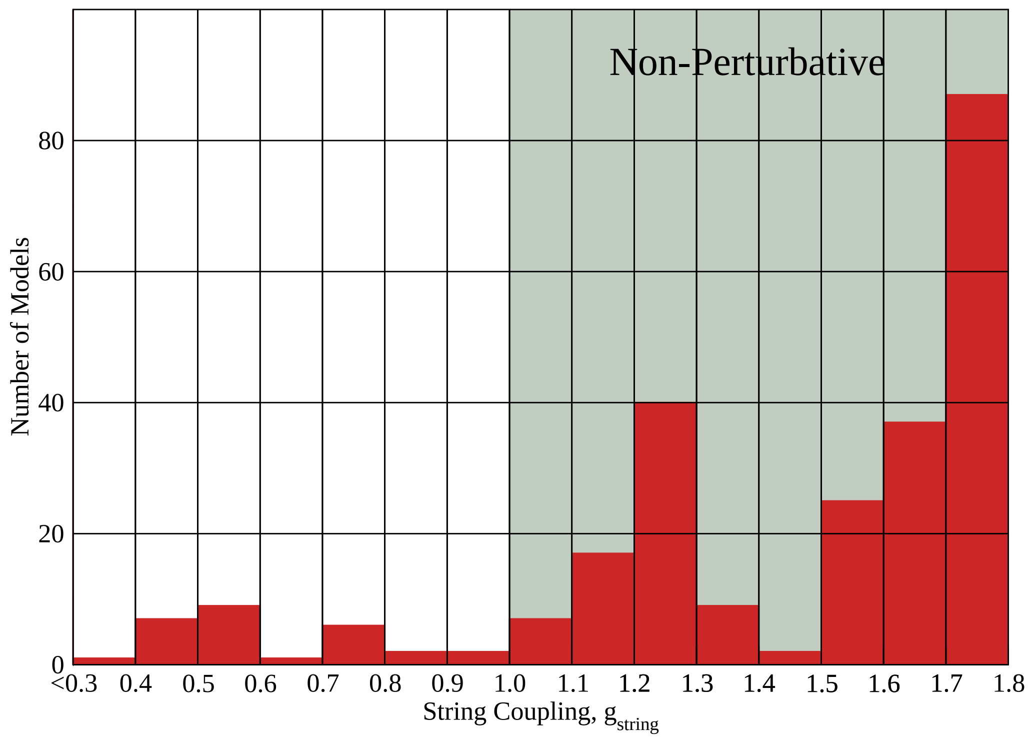

Figure 1: Histogram of the string coupling of the 252 solutions of Reference [6]. Of the 48 models which were not eliminated previously because of dimension six proton decay, 28 have $g_{\text{STRING}} \lesssim 1$.

following useful formula for the proton lifetime:

$$\tau(p \to \pi^0 e^+) \cong 5.21 \times 10^{40} \left(\frac{M_{\mathrm{C}}}{M_{\mathrm{STRING}}} \right)^4 \mathrm{yr}. \tag{8}$$

We can then re-write Equation (6) as

$$g^2_{\mathrm{STRING}} = \alpha_{\mathrm{GUT}} \left(\frac{5.21 \times 10^{40}\ \mathrm{yr}}{\tau(p \to \pi^0 e^+)} \right)^{1/4}. \tag{9}$$

The current (published) limit on the proton lifetime [21]

$$\tau(p \to e^+ + \pi^0) > 1.6 \times 10^{33}\ \mathrm{yr} \tag{10}$$

implies that

$$g^2_{\mathrm{STRING}} \lesssim 600 \frac{M^2_{\mathrm{STRING}}}{M^2_{\mathrm{PL}}}, \tag{11}$$

where we have inserted the definition of α_{GUT} in terms of the Planck scale, Equation (7). If we take a typical value for the string scale $\sim 5.0 \times 10^{17}$ GeV and the Planck scale $\sim 1.2 \times 10^{19}$ GeV , we find that

$$g^2_{\mathrm{STRING}} \lesssim 1. \tag{12}$$

Another interesting point is that the model described in Section 4 of Reference [6] has a small string coupling. There, we found $M_{\mathrm{C}} \sim 2.2 \times 10^{17}$ GeV and $M_{\mathrm{STRING}} \sim 1.0 \times 10^{18}$ GeV . We find $g_{\mathrm{STRING}} \sim 0.5$. This is encouraging because we were able to show that that model is consistent with $F = D = 0$ and the decoupling of unwanted exotics from the low energy spectrum.

Of the 48 solutions which we found in Reference [6], we isolated a handful (9) which exhibited only moderate hierarchies between the scales in the problem. When we look at the string coupling using Equation (6), however, we see that only one of them can be derived from a model at weak coupling. Unsurprisingly, this is also the model with the largest value of M_{C} and thus the longest lived proton.

Note that in all of the models with $g_{\mathrm{STRING}} < 1$, the compactification scale M_{C} is above or equal the 4D GUT scale: see Table 1 on page 10. Hence the threshold corrections in these models, which focus the 3 low energy couplings, come predominantly from the contribution of the exotics with

mass M_{EX}. This result is particularly model dependent. While the KK modes contribute the universal power law running which allows the theory to satisfy the weakly coupled heterotic string boundary condition, Equation (7), they also contribute to the differential running in a way which does not focus the 3 gauge couplings. It is the exotic matter at the intermediate scale which furnishes a contribution to the differential running, allowing for $M_{\text{C}} \gtrsim M_{\text{GUT}}$. However, it is possible that in other string models the KK modes alone would be sufficient to both satisfy the weakly coupled heterotic string boundary condition and focus the 3 gauge couplings.

Finally, we note that the models described in Reference [5] can be interpreted as six dimensional orbifold GUTs. If this is the case, then the relationship in Equation (6) will be amended:

$$g_{\text{STRING}}^2 = 2\frac{M_{\text{STRING}}^2}{M_5 M_6}\alpha_{\text{GUT}} = 16\frac{M_{\text{STRING}}^4}{M_5 M_6 M_{\text{PL}}^2}, \tag{13}$$

where $\ell_{5(6)} \equiv M_{5(6)}^{-1}$ is the radius of the fifth (sixth) direction. In this case, it seems equally likely that a weakly coupled model can be constructed. If we take, for example, $M_5 \sim M_6 \sim M_{\text{C}}$, and the typical value of $M_{\text{STRING}} \sim 5 \times 10^{17}$ GeV , we find $g_{\text{STRING}} \lesssim 1$ requires $M_{\text{C}} \gtrsim 8 \times 10^{16}$ GeV .

Taking more directions larger than the string length pushes us toward stronger and stronger coupling, and it seems likely that if this is the case then some other directions would have to be *smaller* than the string length. This can be seen by looking at the general relationship between g_{STRING} and the other scales in the problem. If we take n extra dimensions to be large, we find

$$g_{\text{STRING}}^2 = 2^{n+2}\frac{M_{\text{STRING}}^{n+2}}{{M_{\text{C}}}^n M_{\text{PL}}^2}. \tag{14}$$

If we take $n = 3$, and a typical string scale, we find that $M_{\text{C}} \gtrsim 3 \times 10^{17}$ GeV .

Conclusions.—In this short note, we have analyzed the string coupling in a class of highly successful models based on anisotropic compactifications of the weakly coupled heterotic string. Of the 252 different solutions consistent with gauge coupling unification found in Reference [6], 48 were not already ruled out by current (dimension six) proton decay bounds. In this paper, out of the 48 solutions (not eliminated by the non-observation of proton decay) we find 28 which are consistent with a weakly coupled heterotic string, $g_{\text{STRING}} < 1$ (see Figure 1).

We also pointed out an interesting correlation between the string scale, the Planck scale, and the compactification scale (which sets the proton lifetime). Specifically, a proton lifetime consistent with current bounds on dimension six operators seems to require weak coupling, for a reasonable choice of parameters. Moreover, we were able to show that one specific (and very well-motivated) example *does* require $g_{\text{STRING}} \sim 0.5$.

For all cases with $g_{\text{STRING}} < 1$, the compactification scale M_{C} is above or equal the 4D GUT scale, $M_{\text{GUT}} \sim 3 \times 10^{16}$ GeV. Hence the threshold corrections in these models, which focus the 3 low energy couplings, come predominantly from the contribution of the exotics with mass M_{EX}. While the KK modes contribute the universal power law running which allows the theory to satisfy the weakly coupled heterotic string boundary condition, Equation (7), they also contribute to the differential running in a way which does not focus the 3 gauge couplings. It is the exotic matter at the intermediate scale which furnishes a contribution to the differential running, allowing for $M_{\text{C}} \gtrsim M_{\text{GUT}}$. This result is model dependent and it is possible that in other string models the KK modes alone would be sufficient to both satisfy the weakly coupled heterotic string boundary condition and focus the 3 gauge couplings.

Finally, we commented on extensions of this work to six (and higher) dimensional orbifold GUTs—barring large threshold corrections from somewhere else (i.e., higher dimensional operators), it seems possible to construct models which are consistent with the weak coupling ansatz in six dimensions. However, in going to higher dimensions, it seems likely that one would have to look for models in which some of the compact directions had radii *smaller than* the string length.

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Table 1: Subset of models listed in Reference [6] which exhibit $g_{\text{STRING}} \lesssim 1$. Note that we define $\vec{n}$ in terms of the brane localized exotics: $\vec{n} \equiv (n_3, n_2, n_1)$, where the following (brane-localized) matter gets mass at the intermediate scale M_{EX} : $n_3 \times \left[(\mathbf{3},1)_{1/3} + (\overline{\mathbf{3}},1)_{-1/3}\right] + n_2 \times [(1,\mathbf{2})_0 + (1,\mathbf{2})_0] + n_1 \times [(1,1)_1 + (1,1)_{-1}]$. (See Reference [6] for more details.)

Bulk Exotics	$\vec{n}$	M_{STRING} in GeV	M_{C} in GeV	M_{EX} in GeV	$\tau(p \to e^+\pi^0)$ in yr	g_{STRING}	α_{GUT}^{-1}
None	$(4,2,0)$	9.18×10^{17}	2.22×10^{17}	4.88×10^{13}	1.77×10^{38}	0.43	22
	$(2,1,0)$	9.18×10^{17}	2.22×10^{17}	2.60×10^{9}	1.77×10^{38}	0.43	22
	$(3,2,3)$	9.88×10^{17}	2.22×10^{17}	2.08×10^{9}	1.32×10^{38}	0.48	19
	$(4,3,6)$	1.08×10^{18}	2.22×10^{17}	1.59×10^{9}	9.23×10^{37}	0.55	16
	$(4,2,1)$	8.26×10^{17}	6.65×10^{16}	5.43×10^{13}	2.19×10^{36}	0.67	27
	$(4,2,2)$	6.87×10^{17}	2.19×10^{16}	6.52×10^{13}	5.34×10^{34}	0.89	39
	$(2,1,1)$	6.87×10^{17}	2.19×10^{16}	6.18×10^{9}	5.34×10^{34}	0.89	39
	$(3,2,4)$	7.07×10^{17}	2.16×10^{16}	5.68×10^{9}	4.52×10^{34}	0.94	37
	$(4,3,7)$	7.28×10^{17}	2.13×10^{16}	5.21×10^{9}	3.79×10^{34}	0.99	35
$\left[(\mathbf{3},1)_{2/3,*} + (\overline{\mathbf{3}},1)_{-2/3,*})\right]^{++} +$ $[(1,\mathbf{2})_{1,*} + (1,\mathbf{2})_{-1,*})]^{--}$	$(4,3,1)$	9.96×10^{17}	7.74×10^{17}	4.50×10^{13}	1.90×10^{40}	0.26	19
	$(4,3,2)$	9.73×10^{17}	2.22×10^{17}	4.61×10^{13}	1.40×10^{38}	0.47	20
	$(2,2,2)$	1.01×10^{18}	2.22×10^{17}	1.92×10^{9}	1.19×10^{38}	0.5	18
	$(3,3,5)$	1.12×10^{18}	2.22×10^{17}	1.43×10^{9}	7.97×10^{37}	0.58	15
	$(4,4,8)$	1.28×10^{18}	2.22×10^{17}	9.64×10^{8}	4.73×10^{37}	0.71	11
	$(3,2,0)$	8.79×10^{17}	6.55×10^{16}	5.10×10^{13}	1.61×10^{36}	0.75	24
	$(4,3,3)$	9.06×10^{17}	6.50×10^{16}	4.95×10^{13}	1.38×10^{36}	0.78	23
$\left[(\mathbf{3},1)_{2/3,*} + (\overline{\mathbf{3}},1)_{-2/3,*})\right]^{--} +$ $[(1,\mathbf{2})_{1,*} + (1,\mathbf{2})_{-1,*})]^{++}$	$(3,1,1)$	1.01×10^{18}	2.22×10^{17}	1.92×10^{9}	1.19×10^{38}	0.5	18
	$(4,2,4)$	1.12×10^{18}	2.22×10^{17}	1.43×10^{9}	7.97×10^{37}	0.58	15
$\left[(\mathbf{3},1)_{2/3,*} + (\overline{\mathbf{3}},1)_{-2/3,*})\right]^{++} +$ $[(1,\mathbf{2})_{1} + (1,\mathbf{2})_{-1})]^{++}$	$(4,2,0)$	9.73×10^{17}	2.22×10^{17}	4.61×10^{13}	1.40×10^{38}	0.47	20
	$(2,1,0)$	1.01×10^{18}	2.22×10^{17}	1.92×10^{9}	1.19×10^{38}	0.5	18
	$(3,2,3)$	1.12×10^{18}	2.22×10^{17}	1.43×10^{9}	7.97×10^{37}	0.58	15
	$(4,3,6)$	1.28×10^{18}	2.22×10^{17}	9.64×10^{8}	4.73×10^{37}	0.71	11
	$(4,2,1)$	9.06×10^{17}	6.50×10^{16}	4.95×10^{13}	1.38×10^{36}	0.78	23
$\left[(\mathbf{3},1)_{2/3,*} + (\overline{\mathbf{3}},1)_{-2/3,*})\right]^{--} +$ $[(1,\mathbf{2})_{1} + (1,\mathbf{2})_{-1})]^{--}$	$(2,1,0)$	9.36×10^{17}	2.22×10^{17}	2.45×10^{9}	1.64×10^{38}	0.45	21
	$(4,2,0)$	9.36×10^{17}	2.22×10^{17}	4.79×10^{13}	1.64×10^{38}	0.45	21
	$(3,2,3)$	1.01×10^{18}	2.22×10^{17}	1.92×10^{9}	1.19×10^{38}	0.5	18
	$(4,3,6)$	1.12×10^{18}	2.22×10^{17}	1.43×10^{9}	7.97×10^{37}	0.58	15
	$(4,2,1)$	8.79×10^{17}	6.55×10^{16}	5.10×10^{13}	1.61×10^{36}	0.75	24

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